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Determinants

This exercise sheet consists of two parts: at first additional exercises are given the solutions of which are provided with the lecture slides and can serve you as further blueprints when solving similar tasks. Then, the actual homework assignments are stated. Please, hand-in your results of the homework assignments through MSTeams at the date and time specified in MSTeams.

Additional Exercises (see the lecture slides for solutions):

Exercise 2.1: Areas of Parallelograms

a) Compute the area A of the parallelogram that is spanned by the two vectors

$$\vec{v} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \qquad \text{and} \qquad \vec{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} .$$

b) Compute the area A of the parallelogram that is given by the four points $O(0,0)$, $A(4,1)$, $B(1,3)$ and $C(5,4)$.

Exercise 2.2: Determinants of 2×2-matrices

a) Compute the values of the following determinants of 2×2-matrices

$$\det \begin{pmatrix} \cos(x) & -\sin(x) \\ \sin(x) & \cos(x) \end{pmatrix} \quad \text{and} \quad \det \begin{pmatrix} \cos(x) & \sin(y) \\ \sin(x) & \cos(y) \end{pmatrix} .$$

b) Solve the following equations involving determinants.

$$\det \begin{pmatrix} 6 & 2 \\ 3 & x \end{pmatrix} = 0 \quad \text{and} \quad \det \begin{pmatrix} 1-\lambda & -2 \\ 2 & 3-\lambda \end{pmatrix} = 0 .$$

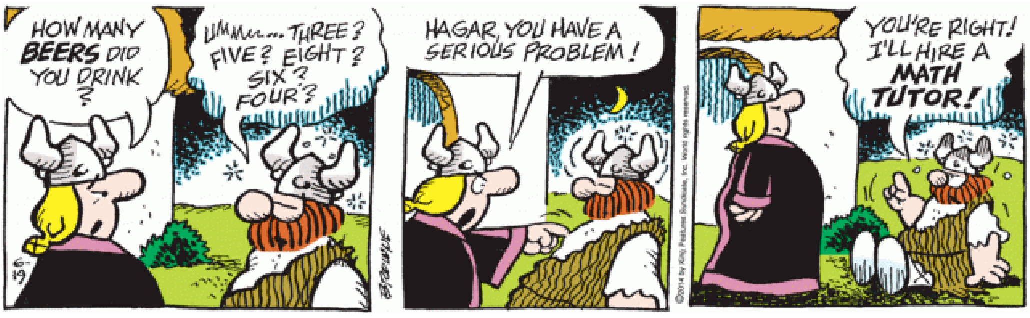
Exercise 2.3: Determinants of 3×3-matrices

a) Compute the values of the following determinants of 3×3-matrices

$$\det \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \det \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} .$$

b) Solve the following equation involving determinants:

$$\det \begin{pmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ x+1 & 4 & x \end{pmatrix} = -2x^3 + 11 .$$



Homework Assignment:

Problem 2.1: Solve the following system of equations given in augmented matrix form and finally check your result by matrix-column multiplication:

$$\left(\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 0 & 300 \\ 1 & 0 & 0 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & -1 & 400 \\ 0 & 1 & -1 & 0 & 0 & -500 \\ 0 & 0 & 1 & -1 & 0 & -100 \end{array}\right)$$

Problem 2.2: Consider the following general system of linear equations in matrix-column form:

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}}_{=:A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}}_{=:x} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_{=:b}.$$

Assume that b is one of the columns of A. What does this imply?

a) In any case $Ax = b$ has a solution.

b) In any case $Ax = b$ has no solution.

c) $Ax = b$ sometimes has a solution and sometimes it has no solution depending on A and b.

Problem 2.3: Compute the following matrix-vector-products

$$\begin{pmatrix} 1 & 1 & -2 \\ 3 & 5 & -1 \\ 1 & 7 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 4 & -2 \\ 3 & 3 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

Problem 2.4: Solve the following equations involving determinants.

a) $\det \begin{pmatrix} 13247 & 13347 \\ 28469 & 28569 \end{pmatrix} = x.$

b) $\det \begin{pmatrix} x-1 & 4 \\ 1 & x+2 \end{pmatrix} = 0.$

c) $\det \begin{pmatrix} x^2+1 & x \\ x\cdot(x-3) & x-2 \end{pmatrix} = 0.$

d) $\det \begin{pmatrix} 2-\lambda & 0 \\ 0 & 7-\lambda \end{pmatrix} = 0.$

Problem 2.5: Compute the values of the following determinants.

a) $\det \begin{pmatrix} 1 & 5 & 8 \\ 40 & -9 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$

b) $\det \begin{pmatrix} 2 & -5 & 9 \\ 5 & -8 & 7 \\ 7 & -13 & 16 \end{pmatrix}.$

c) $\det \begin{pmatrix} 3 & 0 & 5 \\ 0 & 1 & 0 \\ 1 & 0 & 4 \end{pmatrix}.$