PROF. DR. GIORGI CHELIDZE
PROF. DR. MALKHAZ SHASHIASHVILI
PROF. DR. DR. H.C. FLORIAN RUPP

Introduction to Differential Equations

This exercise sheet consists of two parts: at first additional exercises are given the solutions of which are provided with the lecture slides and can serve you as further blueprints when solving similar tasks. Then, the actual homework assignments are stated. Please, hand-in your results of the homework assignments through MSTeams at the date and time specified in MSTeams.

Additional Exercises (see the lecture slides for solutions):

Exercise 5.1: Which of the following equations are separable?

- ☐ $2x(y+1) \cdot x' - y \cdot y' = 0$.
- ☐ $x \cdot x' + \sqrt{a^2 - x^2} \cdot y' = 0$, where a is constant.
- ☐ $x \cdot y' + y \cdot x' + xy(3x' + y') = 0$.
- ☐ $(\mathrm{e}^{2x} + 4) \cdot y' = y$.

Exercise 5.2: Find the general solution of

a) $y \cdot y' = x + 3$.
b) $xy' = y$.

Exercise 5.3: General and specific solutions:

a) Find the general solution of $y' = 3x \cdot \mathrm{e}^{-y}$ and the specific solution that satisfies the condition $y(0) = 1$.
b) Find the specific solution of $y' = \mathrm{e}^{2x+y}$ that has $y = 0$ when $x = 0$.

Exercise 5.4: Solve the equation $y' = (y+1)/(x-1)$ given the boundary condition: $y = 1$ at $x = 0$.

Exercise 5.5: Solve the equation $y' + 2xy = 1$.

Homework Assignment:

Problem 5.1: Use separation of variables to find <u>all</u> solutions of the following four ODEs

$$y'(x) = \frac{\mathrm{d}y(x)}{\mathrm{d}x} = y(x) \cdot (1 + \mathrm{e}^x) , \qquad \text{and} \qquad y'(x) = y(x) \cdot (\sin(x) + \mathrm{e}^x)$$

as well as

$$y'(x) = \frac{y(x)}{4+x} , \qquad \text{and} \qquad y'(x) = \frac{y^2(x) \cdot (x-3)}{x^3} .$$

Problem 5.2: Advertising After initiating an advertising campaign, Andria, the manager of a satellite dish provider in Kutaisi, estimates that t months from now, the number of new subscribers $N(t)$ will be growing at the rate of

$$N'(t) = 154\, t^{2/3} + 37$$

subscribers per month. How many new subscribers should Andria expect 8 months from now?

Problem 5.3: Oil Production – I. A certain oil well that yields 400 barrels of crude oil per month will run dry in 2 years. The price of crude oil is currently 98 USD per barrel and is expected to rise at the constant rate of 40 cents per barrel per month. If the oil is sold as soon as it is extracted from the ground, how much total revenue will be obtained from the well?

Problem 5.4: Oil Production – II. Suppose the owner of the oil well in Problem 5.2 decides to step up production so that 600 barrels per month are extracted but everything else remains the same.

a) How many months pass before the well runs dry?

b) How much total revenue will be obtained from the well?

Problem 5.5: Solve the following initial-value problems.

a) $x^2y' + 2xy = \ln(x)$, with $y(1) = 2$.

b) $xy' = y + x^2\sin(x)$, with $y(\pi) = 0$.

Problem 5.6: Domar's capital expansion model is

$$I'(t) \;=\; hkI(t)\,,$$

where $I(t)$ is the investment, h is the investment productivity (constant), k is the marginal productivity to the consumer (constant), and t is the time.

a) Use separation of variables to solve the differential equation.

b) Rewrite the solution in terms of the initial condition $I_0 = I(0)$.

Problem 5.7: The marginal revenue for a certain product is given by $R'(x) = 300 - 2x$. Find the total-revenue function $R(x)$ assuming that $R(0) = 0$.

Problem 5.8: The marginal cost for a certain product is given by $C'(x) = 2.6 - 0.02x$. Find the total-cost function $C(x)$ and the average cost $A(x)$ assuming that fixed costs are 120 GEL; i.e. $C(0) = 120$.

Problem 5.9: The reaction R in pleasure units by a consumer receiving S units of a product can be modeled by the differential equation

$$\frac{\mathrm{d}R(S)}{\mathrm{d}S} \;=\; \frac{k}{S+1}\,,$$

where k is a positive constant.

a) Use separation of variables to solve the differential equation.

b) Rewrite the solution in terms of the initial condition $R(0) = 0$.

c) Explain why the condition $R(0) = 0$ is reasonable