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Optimization & The Method of Least Squares

This exercise sheet consists of two parts: at first additional exercises are given the solutions of which are provided with the lecture slides and can serve you as further blueprints when solving similar tasks. Then, the actual homework assignments are stated. Please, hand-in your results of the homework assignments through MSTeams at the date and time specified in MSTeams.

Additional Exercises (see the lecture slides for solutions):

Exercise 11.1: Paul Johnson is a salesman whose territory borders on a lake and can be described in terms of a rectangular grid as the region bounded by the curve $y = x^2$ (the lakefront) and the lines $y = 0$ and $x = 3$ as shown in the figure, where x and y are in miles. He determines that the number of units $S(x, y)$ he can sell at each grid point (x, y) in his region is given by the function

$$S(x, y) \;=\; 4x^2 - 16x + 4y^2 - 4y + 20\,.$$

At what point(s) in his sales territory should Paul expect maximum sales to occur, and what are his maximum expected sales? Answer the same question for minimum sales.

Exercise 11.2: Determine the least-squares line $y = mx + b$ that best fits the observation

x	0	200	600	1000	1200
y	150	100	70	20	30

Exercise 11.3: A manufacturer gathers the data listed in the accompanying table relating the level of production x (hundred units) of a particular commodity to the demand price p (GEL per unit) at which all the units will be sold:

Production x	5	10	15	20	25	30	35
Demand price p	44	38	32	25	18	12	6

a) Plot these data on a graph.

b) Find the equation of the least-squares line for the data and plot this least-squares line into the sketch of part **a)**.

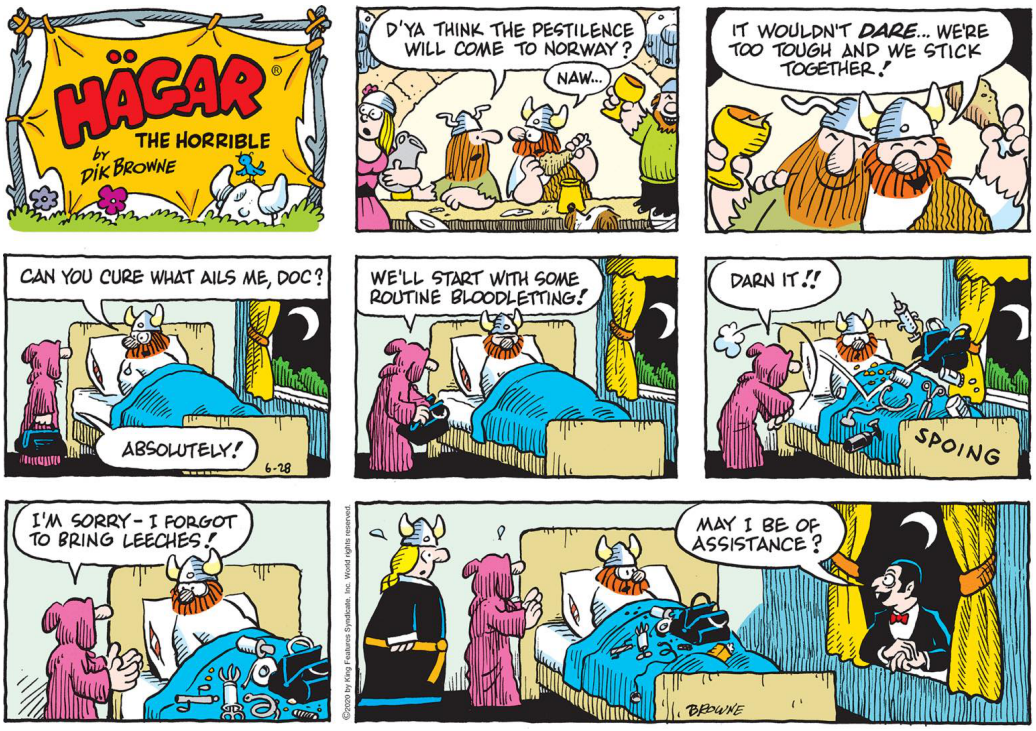
c) Use the linear demand equation you found in part **b)** to predict the revenue the manufacturer should expect if 4000 units ($x = 40$) are produced.

Homework Assignment:

Problem 11.1: Find all interior and boundary critical points and determine the largest and smallest values of the function $f(x, y)$ over the given closed, bounded region R.

a) $f(x, y) = xy - x - 3y$ on the triangular region R with vertices $(0, 0)$, $(5, 0)$, $(5, 5)$.

b) $f(x, y) = xy^2$ on the quarter circular region R bounded by $x^2 + y^2 = 12$ with $x \geq 0$, $y \geq 0$.



Problem 11.2: A farmer has 300 km^2 on which to plant two crops, celery and lettuce. Each acre of celery costs 250 GEL to plant and tend, and each acre of lettuce costs 300 GEL to plant and tend. The farmer has 81000 GEL available to cover these costs.

a) Suppose the farmer makes a profit of 45 GEL per km^2 of celery and 50 GEL per km^2 of lettuce. Write the profit function, determine how many km^2 of celery and lettuce he should plant to maximize profit, and state the maximum profit. (*Hint*: Since the graph of the profit function is a plane, you will not need to check the interior for possible critical points.)

b) Suppose the farmer's profit function is instead $P(x, y) = -x^2 - y^2 + 600y - 75000$. Assuming the same constraints, how many acres of celery and lettuce should he plant to maximize profit, and what is that maximum profit?

Problem 11.3: In each case, give the least-squares line $y = mx + b$ that best fits through the observed data points (x, y) and draw the data points together with the least-squares line.

a)

x	1	2	4	5
y	1	3	3	4

b)

x	1	3	5
y	2	4	7

c)

x	1	2	3	5
y	0	1	3	4

d)

x	1	2	4
y	3	5	8

Problem 11.4: Hagar the Horrible is listing his annual turn-overs from plundering (in units of 1 million gold coins) for his first 5 years of operation with his 'new' crew. He obtains the accompanying table:

Year	1	2	3	4	5
Turn-over	0.9	1.5	1.9	2.4	3.0

a) Plot these data on a graph.

b) Find the equation of the least-squares line for the data and plot this least-squares line into the sketch of part **a)**.

c) Use the least-squares line **b)** to predict Hagar's turn-over in the next, the sixth, year.