PROF. DR. GIORGI CHELIDZE
PROF. DR. MALKHAZ SHASHIASHVILI
PROF. DR. DR. H.C. FLORIAN RUPP

Double Integrals

This exercise sheet consists of two parts: at first additional exercises are given the solutions of which are provided with the lecture slides and can serve you as further blueprints when solving similar tasks. Then, the actual homework assignments are stated. Please, hand-in your results of the homework assignments through MSTeams at the date and time specified in MSTeams.

Additional Exercises (see the lecture slides for solutions):

Exercise 13.1: (Rectangular regions)

a) Let R be the rectangular region $1 \leq x \leq 2$, $0 \leq y \leq \pi$. Evaluate the double integral

$$\iint_R y \cdot \sin(xy)\,\mathrm{d}A\,.$$

b) Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the three coordinate planes.

c) Let R be the rectangular region $0 \leq x \leq \frac{1}{2}\pi$, $0 \leq y \leq \frac{1}{2}\pi$. Evaluate the double integral

$$\iint_R \sin(x) \cdot \cos(y)\,\mathrm{d}A\,.$$

Exercise 13.2: Let R be bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$. Evaluate the double integral

$$\iint_R (x + 2y)\,\mathrm{d}A\,.$$

Exercise 13.3: Find the volume of the solid S that lies under the paraboloid $z = x^2 + y^2$ and above the region R in the x-y-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

Exercise 13.4: Let R be the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. Evaluate the double integral

$$\iint_R xy\,\mathrm{d}A\,.$$

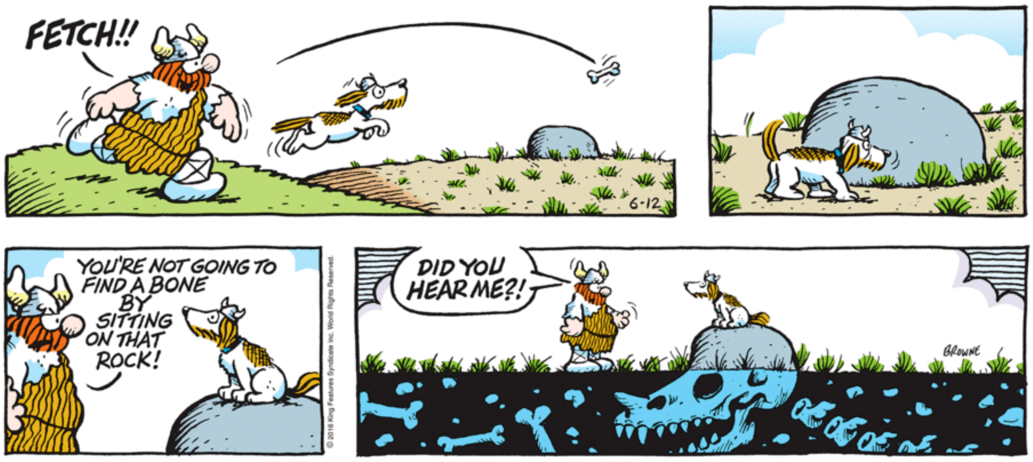
Homework Assignment:

Problem 13.1: Evaluate the double integrals

$$\int_0^1 \int_1^2 x^2 y\,\mathrm{d}x\,\mathrm{d}y\,, \qquad \text{and} \qquad \int_0^1 \int_1^5 y \cdot \sqrt{1 - y^2}\,\mathrm{d}x\,\mathrm{d}y\,.$$

Problem 13.2: Sketch the region of integration for the given integral and set up an equivalent integral with the order of integration reversed:

$$\int_0^2 \int_0^{4-x^2} f(x,y)\,\mathrm{d}y\,\mathrm{d}x\,, \qquad \text{and} \qquad \int_{-1}^1 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$



Problem 13.3: Evaluate the double integral over the specified region R. Choose the order of integration carefully.

a) $\iint\limits_R x^3 e^{x^2 y} \mathrm{d}A$, with $0 \le x \le 1$ and $0 \le y \le 1$.

b) $\iint\limits_R e^{x^3} \mathrm{d}A$, with $\sqrt{y} \le x \le 1$ and $0 \le y \le 1$.

c) $\iint\limits_R y^2 \mathrm{d}A$, where R is the triangular region with vertices $(0,1)$, $(1,2)$, and $(4,1)$.

d) $\iint\limits_R (2x - y) \mathrm{d}A$, where R is bounded by the circle with center the origin and radius 2.

Problem 13.4: In each case, find the averge value of the function $f(x,y)$ over the region R.

a) $f(x,y) = xy$, where R is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,3)$.

b) $f(x,y) = x \cdot \sin(y)$, where R is enclosed by the curves $y = 0$, $y = x^2$, and $x = 1$.

Problem 13.5: A bicycle dealer has found that if 10-speed bicycles are sold for x GEL apiece and the price of gasoline is y cents per liter, then approximately

$$Q(x,y) \;=\; 200 - 24\sqrt{x} + 4 \cdot (0.1y + 3)^{3/2}$$

bicycles will be sold each month. If the price of bicycles varies between 289 GEL and 324 GEL during a typical month, and the price of gasoline varies between 2.96 and 3.05, approximately how many bicycles will be sold each month on average?

Problem 13.6: A community is laid out as a rectangular grid in relation to two main streets that intersect at the city center. Each point in the community has coordinates (x,y) in this grid, for $-10 \le x \le 10$, $-8 \le y \le 8$ with x and y measured in kilometers. Suppose the value of the land located at the point (x,y) is V thousand GEL, where

$$V(x,y) \;=\; (250 + 17x)e^{-0.01x - 0.05y}\,.$$

Estimate the value of the block of land occupying the rectangular region $1 \le x \le 3$ and $0 \le y \le 2$.

Problem 13.7: The population density is $\rho(x,y) = 2500e^{-0.01x-0.02y}$ people per square kilometer at each point (x,y) within the triangular region R with vertices $(-5,-2)$, $(0,3)$, and $(5,-2)$. Find the total population in the region R.